

University of Saskatchewan  
Department of Physics and Engineering Physics  
EP 228.3  
Final Examination

*A CLOSED BOOK EXAMINATION*  
[Examiner D.A. Degenstein]

Time: 3 hours.

Date: April, 2005

Instructions: Candidates are to answer **ALL** questions in the booklets provided.

There are 13 questions in the exam.

All questions **DO NOT** have the same value.

Electronic calculators are required.

One formula sheet is allowed.

Please read each question carefully before attempting it.

**THINK** before you act!!!!

- 1) Express each of the complex expressions in terms of  $z = x + iy$ . Show some work or you get no marks. (**12 marks**)

i)  $z = \frac{dz_1}{dt}$  where  $z_1 = 5 - i2t$

ii)  $z = \frac{(z_1^* - z_2)}{z_2}$  where  $z_1 = 2e^{i\frac{4\pi}{3}}$  and  $z_2 = 2e^{-i\frac{\pi}{3}}$

iii)  $z(3,0)$  where  $z(x,t) = \frac{d^2 e^{z_1}}{dx^2}$  and  $z_1 = -i\frac{\pi}{3}t - 2x$

iv)  $z = z_1 z_2$  where  $z_1 = (8 + i12)^{-1}$  and  $z_2 = 2 - i3$

- 2) Show, using the complex representation of  $\cos\theta$  and  $\sin\theta$ , that: (**5 marks**)

$$[\tan(\theta_1) - \tan(\theta_2)]\cos(\theta_1)\cos(\theta_2) = \sin(\theta_1 - \theta_2)$$

- 3) What are the non-zero frequency components,  $a_n$  and  $b_n$  values, and the angular frequencies of these components for the function

$$f(t) = 6[\tan(60\pi t) - \tan(20\pi t)]\cos(60\pi t)\cos(20\pi t) - 5\cos(40\pi t) + 1.2\sin(40\pi t) - 3.8$$

on the interval  $[-0.1, 0.1]$ ? (**5 marks**)

- 4) Assume the number of Uranium molecules in an experiment,  $N_t$ , can be defined at any time as  $N_t = N_{t-1}r - A$ . In this expression  $A$  and  $r$  are constants and  $N_{t-1}$  is the number of Uranium molecules at the previous time step. Prove by induction that the number of Uranium molecules in the system at any time  $t$  is given by

$$N_t = N_0 r^t - A \frac{(1 - r^t)}{(1 - r)}$$

where  $N_0$  is the number of molecules at time  $t = 0$  s. (5 marks)

- 5) What are the Fourier coefficients,  $a_n$  and  $b_n$ , for the function shown in Figure 5.1? Assume  $T_0 = -3.0$  s and  $T_f = 3.0$  s. (8 marks)

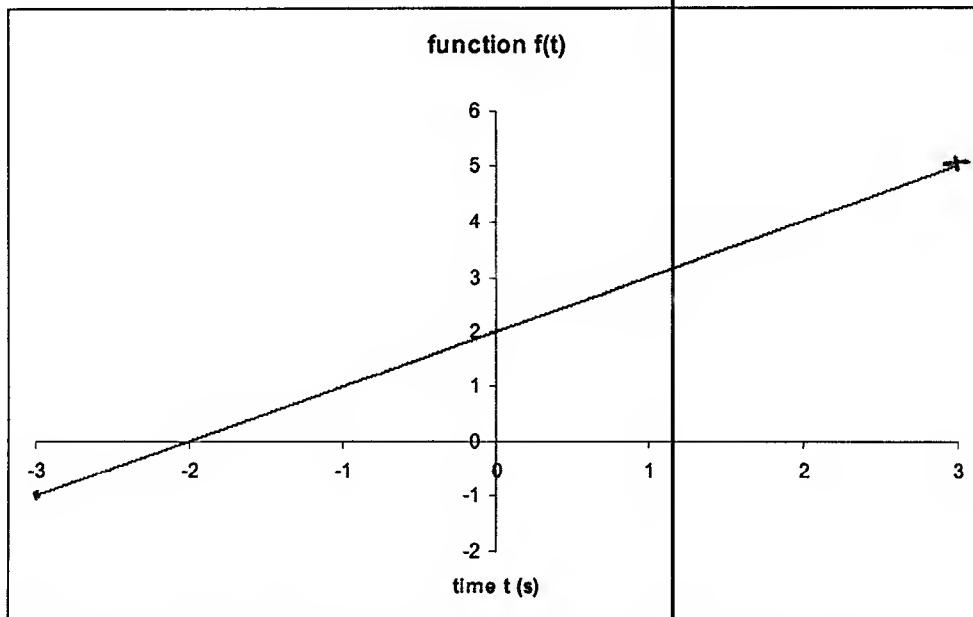


Figure 5.1: Estimate the required points by reading them from the graph.

- 6) Given the matrix  $A = \omega \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$  and the definition  $e^{At} = I + \frac{At}{1!} + \frac{A^2 t^2}{2!} + \frac{A^3 t^3}{3!} + \frac{A^4 t^4}{4!} + \dots$  show that (6 marks)

$$e^{At} = \begin{pmatrix} \cos(\omega t) & -\sin(\omega t) \\ \sin(\omega t) & \cos(\omega t) \end{pmatrix}.$$

- 7) At a height  $h$  above the surface of the Earth the gravitational force on a mass  $m$  is given by

$$F = \frac{mMG}{(R+h)^2} \quad (7.1)$$

where  $M$  and  $R$  are respectively the mass and radius of the Earth and  $G$  is Newton's gravitational constant. For small values of  $h$  it is common to use the approximation

$F = mg$  where  $g = \frac{MG}{R^2}$ . If more precision is required for a particular calculation

what are the next two correction terms in the Taylor series expansion of equation 7.1.  
**(5 marks)**

- 8) Given the matrix  $A = \begin{pmatrix} -3 & -1 \\ 1 & 0 \end{pmatrix}$ : **(10 marks)**

i) What is the determinant of  $A$ ?

ii) What is the inverse of  $A$ ?

iii) What is  $x$  in  $Ax = \begin{pmatrix} 2 \\ i \end{pmatrix}$ ?

iv) What are the eigenvalues of  $A$ ?

v) What are the eigenvectors of  $A$ ?

- 9) Assume  $p = (-5, 2)$  is a point in a two-dimensional Cartesian coordinate system represented by the two unit vectors  $(\hat{x}, \hat{y})$  then: **(9 marks)**

i) Given the vectors  $\bar{v}_1 = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$  and  $\bar{v}_2 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$  what are the constants that multiply  $\bar{v}_1$  and  $\bar{v}_2$  such that the point  $p$  is a linear combination of  $\bar{v}_1$  and  $\bar{v}_2$ ?

ii) Given the vectors  $\bar{v}_1 = \begin{pmatrix} -\frac{\sqrt{2}}{2} \\ \frac{2}{\sqrt{2}} \end{pmatrix}$  and  $\bar{v}_2 = \begin{pmatrix} -\frac{\sqrt{2}}{2} \\ -\frac{2}{\sqrt{2}} \end{pmatrix}$  what are the constants that multiply  $\bar{v}_1$  and  $\bar{v}_2$  such that the point  $p$  is a linear combination of  $\bar{v}_1$  and  $\bar{v}_2$ ?

iii) The vectors  $\bar{v}_1$  and  $\bar{v}_2$  in part ii) represent a coordinate system that is rotated by a particular angle with respect to the original  $(\hat{x}, \hat{y})$  coordinate system. What is the value of the vector  $p$ , in the  $(\hat{x}, \hat{y})$  coordinate system, if it is rotated by the same amount as  $\bar{v}_1$  and  $\bar{v}_2$ ?

$$x^3 + 3x^2(-3) + 3x(9) + (-3)$$

- 10) Take a Taylor Series expansion of  $f(x) = -2 + 3(x - 3) - 8(x - 3)^2 + 2.5(x - 3)^3$  about the point  $x = 3$ . What are the first three non-zero coefficients of this expansion? What is the value of the MacLaurin series expansion of this function evaluated at  $x = 4$ ? (5 marks)

- 11) Given the following data set : (9 marks)

X	f(x)
-10	-26
-5	14
0	4
5	-56
10	-166

Table 11.1

- Find the coefficients of the quadratic  $f(x) = a_2x^2 + a_1x + a_0$  that goes through the two end-points and the middle point of this data set.
- What are the values of this quadratic at  $x = -5$  and  $x = 5$ ?
- What are the best approximations for the numerical derivatives at each of the relevant data points?

- 12) If  $z_1$  and  $z_2$  are two complex numbers, as seen in Figure 12.1 below, show that  $z_1 z_2^* + z_1^* z_2 = 2|z_1||z_2|\cos\theta$ . Also, use this result and Figure 12.2 to prove,  $|z_3|^2 = |z_1|^2 + |z_2|^2 - 2|z_1||z_2|\cos\theta$ , the Law of Cosines. (6 marks)

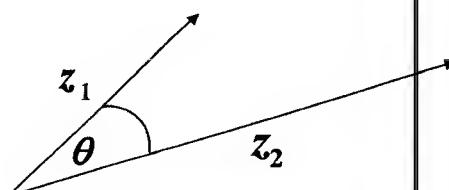


Figure 12.1

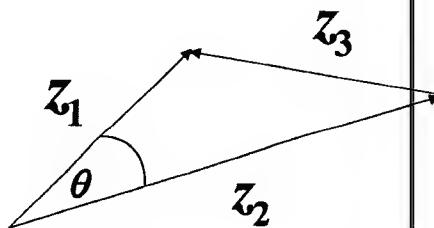


Figure 12.2

13) Given the following data :

$x$	$f(x)$	$f'(x)$
2	9	
3		6
4		1
5		-7
6		-1
7	13	

where the derivative was taken using the central difference method what is the original data set? (**5 marks**)

- End of Examination -